

# On The Distributive Law

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The distributive law is a powerful tool in mathematics. However, it is important to know when to use it, and when not to use it.

## Some Things are Distributive

- $x(y + z) = xy + xz$ , allowing maneuvers such as  $2(y + z) = 2y + 2z$ , or  $-(y + z) = -y - z$ .
- $\frac{(y+z)}{x} = \frac{y}{x} + \frac{z}{x}$ , allowing maneuvers such as  $\frac{(y+z)}{3} = \frac{y}{3} + \frac{z}{3}$ .

## Others Have Alternative Formulae

- $(x + y)^2 \neq x^2 + y^2$  but rather  $(x + y)^2 = x^2 + 2xy + y^2$ .
- $(x + y)^3 \neq x^3 + y^3$  but rather  $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ .
- $(x + y)^4 \neq x^4 + y^4$  but rather  $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ .
- $(x + y)^{-1} \neq x^{-1} + y^{-1}$  but rather  $(x + y)/xy$ .
- $\sin(x + y) \neq (\sin x) + (\sin y)$  but rather  $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ .
- $\cos(x + y) \neq (\cos x) + (\cos y)$  but rather  $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ .
- $e^{x+y} \neq e^x + e^y$  but rather  $e^{x+y} = e^x e^y$ .
- Most interestingly,  $\tan(x + y) \neq (\tan x) + (\tan y)$  but rather

$$\tan(x + y) = \frac{\sin(x)\cos(y) + \cos(x)\sin(y)}{\cos(x)\cos(y) - \sin(x)\sin(y)} = \frac{\cot(y) + \cot(x)}{\cot(x)\cot(y) - 1} = \frac{\tan(x) + \tan(y)}{\tan(y)\tan(x) - 1}$$

## Others Have No Easy Form

- $\log(x + y) \neq \log(x) + \log(y)$  but rather  $\log(xy) = (\log x) + (\log y)$ .
- $\ln(x + y) \neq \ln(x) + \ln(y)$  but rather  $\ln(xy) = (\ln x) + (\ln y)$ .
- $\sqrt{x + y} \neq \sqrt{x} + \sqrt{y}$  but rather  $\sqrt{xy} = (\sqrt{x})(\sqrt{y})$ .
- $\sqrt[3]{x + y} \neq \sqrt[3]{x} + \sqrt[3]{y}$  but rather  $\sqrt[3]{xy} = (\sqrt[3]{x})(\sqrt[3]{y})$ .
- $(ab)! \neq (a!)(b!)$  and has no easy form.

The statements on this page are intended to apply to all  $x, y, z$  in the real numbers. For example, obviously  $(x + y)^4 = x^4 + y^4$  if either  $x$  or  $y$  are zero.